

Further Maths A Level

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You will be given chapters 1, 2, and 3 from the pure textbook, don't worry this is mainly higher GCSE work to ensure you have the basics in place and to ensure in September you are able to hit the ground running.

Complete each of the mixed exercises on separate paper – so they can be handed in individually We will collect these from you in the first lesson of further maths next year – please don't disappoint us by forgetting to bring it – not a good first impression!

They will be marked, and you will receive feedback so that you are able to complete any corrections

This is your first chance to impress us and convince us that you have further maths potential

We will be completing chapter assessments to make sure your skills are up to scratch in the first week.

To support you we have included 3 PowerPoints which are there as a guide to work through the different chapters with lots of opportunity for self assessment. We have also given additional youtube resources on the next slide should you need any more support.

We look forward to seeing all of your hard work in September.

Additional Resources:

- Bicen Maths, detailed tutorials on each chapter
 <u>https://www.youtube.com/@BicenMaths/playlists</u>
- Pete Hart, detailed tutorials on each chapter <u>https://www.youtube.com/user/pjhart1893/playlists</u>
- <u>E</u>xam solutions is a fantastic resource for additional video support on each chapter as well as exam question practice. https://www.examsolutions.net/a-level-maths/edexcel/

cos2 x - sin



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Task 1:

Complete each of these mixed exercises on separate paper – so they can be handed in individually. We will collect these from you in the first lesson of further maths next year – please don't disappoint us by forgetting to bring it – not a good first impression! They will be marked, and you will receive feedback so that you are able to complete any corrections

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	1 Simplify: a $y^3 \times y^5$	$3x^2 \times 2x^5$	c $(4x^2)^3 \div 2x^5$	d $4b^2 \times 3b^3 \times b^4$
	2 Expand and sim a $(x+3)(x-5)$		(-7)(3x+1) c $(2x+1)$	5)(3x - y + 2)
	1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1	b (<i>x</i> +	(-2)(x-3)(x+7) c $(2x+$	3)(x-2)(3x-1)
	4 Expand the brac a $3(5y+4)$		c $5x(2x+3) - 2x(1-3x)$	d $3x^2(1+3x) - 2x(3x-2)$
5	Factorise these expr a $3x^2 + 4x$		c $x^2 + xy + xy^2$	d $8xy^2 + 10x^2y$
6	Factorise: a $x^2 + 3x + 2$ e $5x^2 - 13x - 6$		c $x^2 - 2x - 35$	d $2x^2 - x - 3$
7	Factorise: a $2x^3 + 6x$	b $x^3 - 36x$	c $2x^3 + 7x^2 - 15x$	
8	Simplify: a $9x^3 \div 3x^{-3}$	b $(4^{\frac{3}{2}})^{\frac{1}{3}}$	c $3x^{-2} \times 2x^4$	d $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$
9	Evaluate: a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$	b $\left(\frac{225}{289}\right)^{\frac{3}{2}}$		
10	Simplify			

10 Simplify:

a
$$\frac{3}{\sqrt{63}}$$
 b $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

- **11 a** Find the value of $35x^2 + 2x 48$ when x = 25.
 - **b** By factorising the expression, show that your answer to part **a** can be written as the product of two prime factors.

- 12 Expand and simplify if possible:
 - **a** $\sqrt{2}(3+\sqrt{5})$ **b** $(2-\sqrt{5})(5+\sqrt{3})$ **c** $(6-\sqrt{2})(4-\sqrt{7})$
- 13 Rationalise the denominator and simplify:
 - **a** $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2}-1}$ **c** $\frac{3}{\sqrt{3}-2}$ **d** $\frac{\sqrt{23}-\sqrt{37}}{\sqrt{23}+\sqrt{37}}$ **e** $\frac{1}{(2+\sqrt{3})^2}$ **f** $\frac{1}{(4-\sqrt{7})^2}$
- 14 a Given that $x^3 x^2 17x 15 = (x + 3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c.
 - **b** Hence, fully factorise $x^3 x^2 17x 15$.

E 15 Given that $y = \frac{1}{64}x^3$ express each of the following in the form kx^n , where k and n are constants. **a** $y^{\frac{1}{3}}$ **b** $4y^{-1}$ (1 mark)

- (E/P) 16 Show that $\frac{5}{\sqrt{75} \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. (5 marks)
 - (2 marks) (2 marks) (5 $\sqrt{11}$).
- (3 marks) (3 marks)
- **E/P** 19 Express 27^{2x+1} in the form 3^y , stating y in terms of x. (2 marks)
- (E/P) 20 Solve the equation $8 + x\sqrt{12} = \frac{8x}{\sqrt{3}}$ Give your answer in the form $a\sqrt{b}$ where a and b are integers. (4 marks)
- P 21 A rectangle has a length of (1 + √3) cm and area of √12 cm². Calculate the width of the rectangle in cm. Express your answer in the form a + b√3, where a and b are integers to be found.
- (2 marks) 22 Show that $\frac{(2-\sqrt{x})}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} 4 + x^{\frac{1}{2}}$. (2 marks)
- **E/P** 23 Given that $243\sqrt{3} = 3^a$, find the value of *a*. (3 marks)

(E/P) 24 Given that $\frac{4x^3 + x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$, write down the value of *a* and the value of *b*. (2 marks)

Mixed exercise 2

- 1 Solve the following equations without a calculator. Leave your answers in surd form where necessary. **a** $y^2 + 3y + 2 = 0$ **b** $3x^2 + 13x - 10 = 0$ **c** $5x^2 - 10x = 4x + 3$ **d** $(2x - 5)^2 = 7$
- 2 Sketch graphs of the following equations: **a** $y = x^2 + 5x + 4$ **b** $y = 2x^2 + x - 3$ **c** $y = 6 - 10x - 4x^2$ **d** $y = 15x - 2x^2$
- **E** 3 $f(x) = x^2 + 3x 5$ and g(x) = 4x + k, where k is a constant.
 - **a** Given that f(3) = g(3), find the value of k. (3 marks)
 - **b** Find the values of x for which f(x) = g(x). (3 marks)
 - 4 Solve the following equations, giving your answers correct to 3 significant figures: **a** $k^2 + 11k - 1 = 0$ **b** $2t^2 - 5t + 1 = 0$ **c** $10 - x - x^2 = 7$ **d** $(3x - 1)^2 = 3 - x^2$

5 Write each of these expressions in the form $p(x+q)^2 + r$, where p, q and r are constants to be found: **a** $x^2 + 12x - 9$ **b** $5x^2 - 40x + 13$ **c** $8x - 2x^2$ **d** $3x^2 - (x+1)^2$

6 Find the value k for which the equation $5x^2 - 2x + k = 0$ has exactly one solution. (2 marks)

E 7 C	Given that	for all val	lues of x:
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a

	$3x^2 + 12x + 5 = p(x+q)^2 + r$	
ı	find the values of p , q and r .	(3 marks)

b Hence solve the equation $3x^2 + 12x + 5 = 0$.	(2 marks)
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8	The function f is defined as $f(x) = 2^{2x} - 20(2^x) + 64$, $x \in \mathbb{R}$.	
	a Write $f(x)$ in the form $(2^x - a)(2^x - b)$, where a and b are real constants.	(2 marks)
	b Hence find the two roots of $f(x)$.	(2 marks)

9 Find, as surds, the roots of the equation:

$$2(x+1)(x-4) - (x-2)^2 = 0.$$

- **10** Use algebra to solve (x 1)(x + 2) = 18.
- (E/P) 11 A diver launches herself off a springboard. The height of the diver, in metres, above the pool t seconds after launch can be modelled by the following function:

		$h(t) = 5t - 10t^2 + 10, t \ge 0$	
	a	How high is the springboard above the water?	(1 mark)
	b	Use the model to find the time at which the diver hits the water.	(3 marks)
	c	Rearrange $h(t)$ into the form $A - B(t - C)^2$ and give the values of the constants A , B and C .	(3 marks)
	d	Using your answer to part c or otherwise, find the maximum height of the diver, a	and
		the time at which this maximum height is reached.	(2 marks)
E/P	12	 For this question, f(x) = 4kx² + (4k + 2)x + 1, where k is a real constant. a Find the discriminant of f(x) in terms of k. b By simplifying your answer to part a or otherwise, prove that f(x) has two distinct real roots for all non-zero values of k. 	(3 marks) (2 marks)
		c Explain why $f(x)$ cannot have two distinct real roots when $k = 0$.	(1 mark)
E/P	13	Find all of the roots of the function $r(x) = x^8 - 17x^4 + 16$.	(5 marks)
E/P	14	Lynn is selling cushions as part of an enterprise project. On her first attempt, she sold cushions at the cost of £15 each. She hopes to sell more cushions next time. Her advise	

- suggests that she can expect to sell 10 more cushions for every £1 that she lowers the price. **a** The number of cushions sold c can be modelled by the equation c = 230 - Hp, where
 - $\pounds p$ is the price of each cushion and H is a constant. Determine the value of H. (1 mark) To model her total revenue, £r, Lynn multiplies the number of cushions sold by the price of

each cushion. She writes this as r = p(230 - Hp).

- **b** Rearrange r into the form $A B(p C)^2$, where A, B and C are constants to be found. (3 marks)
- c Using your answer to part b or otherwise, show that Lynn can increase her revenue by £122.50 through lowering her prices, and state the optimum selling price of a cushion. (2 marks)

$ 1 2kx - y = 4 \\ 4kx + 3y = -2 $	
are two simultaneous equations, where k is a constant.	
a Show that $y = -2$.	(3 marks)
b Find an expression for x in terms of the constant k .	(1 mark)
2 Solve the simultaneous equations: x + 2y = 3	
$x^2 - 4y^2 = -33$	(7 marks)
 B Given the simultaneous equations x - 2y = 1 3xy - y² = 8 a Show that 5y² + 3y - 8 = 0. b Hence find the pairs (x, y) for which the simultaneous equations are satisfied. 	(2 marks) (5 marks)
	(/
E 4 a By eliminating y from the equations $x + y = 2$ $x^{2} + xy - y^{2} = -1$ show that $x^{2} - 6x + 3 = 0$.	(2 marks)
b Hence, or otherwise solve the simultaneous equations	
x + y = 2	
$x^2 + xy - y^2 = -1$	
giving x and y in the form $a \pm b\sqrt{6}$, where a and b are integers.	(5 marks)
5 a Given that $3^x = 9^{y-1}$, show that $x = 2y - 2$.	(1 mark)
b Solve the simultaneous equations:	
x = 2y - 2	
$x^2 = y^2 + 7$	(6 marks)
) 6 Solve the simultaneous equations: x + 2y = 3	
$x^2 - 2y + 4y^2 = 18$	(7 marks)
7 The curve and the line given by the equations $kx^{2} - xy + (k + 1)x = 1$ $-\frac{k}{2}x + y = 1$	
where k is a non-zero constant, intersect at a single point.	
a Find the value of k.	(5 marks)
b Give the coordinates of the point of intersection of the line and the curve.	(3 marks)

P	8	A person throws a ball in a sports hall. The height of the ball, <i>h</i> m, can be modelled in relation to the horizontal distance from the point it was thrown from by the quadratic equation: $h = -\frac{3}{10}x^2 + \frac{5}{2}x + \frac{3}{2}$	
		The hall has a sloping ceiling which can be modelled with equation $h = \frac{15}{2} - \frac{1}{5}x.$	X
		$n = \frac{1}{2} - \frac{1}{5}x$. Determine whether the model predicts that the ball will hit the ceiling.	(5 marks)
2	0		(0 111113)
Ð	,	Give your answers in set notation. a Solve the inequality $3x - 8 > x + 13$.	(2 marks)
		b Solve the inequality $x^2 - 5x - 14 > 0$.	(4 marks)
E)	10	Find the set of values of x for which $(x - 1)(x - 4) < 2(x - 4)$.	(6 marks)
5		a Use algebra to solve $(x - 1)(x + 2) = 18$.	(2 marks)
9		 b Hence, or otherwise, find the set of values of x for which (x - 1)(x + 2) > 18. Give your answer in set notation. 	(2 marks)
	12	Find the set of values of x for which:	
		a $6x - 7 < 2x + 3$	(2 marks)
		b $2x^2 - 11x + 5 < 0$	(4 marks)
		$c 5 < \frac{20}{x}$	(4 marks)
		d both $6x - 7 < 2x + 3$ and $2x^2 - 11x + 5 < 0$.	(2 marks)
E	13	Find the set of values of x that satisfy $\frac{8}{x^2} + 1 \le \frac{9}{x}$, $x \ne 0$	(5 marks)
E	14	Find the values of k for which $kx^2 + 8x + 5 = 0$ has real roots.	(3 marks)
/P)	15	The equation $2x^2 + 4kx - 5k = 0$, where k is a constant, has no real roots.	
_		Prove that k satisfies the inequality $-\frac{5}{2} < k < 0$.	(3 marks)
E	16	a Sketch the graphs of $y = f(x) = x^2 + 2x - 15$ and $g(x) = 6 - 2x$ on the same axes.	(4 marks)
0		b Find the coordinates of any points of intersection.	(3 marks)
		c Write down the set of values of x for which $f(x) > g(x)$.	(1 mark)
E	17	Find the set of values of x for which the curve with equation $y = 2x^2 + 3x - 15$ is below the line with equation $y = 8 + 2x$.	(5 marks)
E	18	On a coordinate grid, shade the region that satisfies the inequalities: $y > x^2 + 4x - 12$ and $y < 4 - x^2$.	(5 marks)
/P)	19	a On a coordinate grid, shade the region that satisfies the inequalities	
		y + x < 6, $y < 2x + 9$, $y > 3$ and $x > 0$.	(6 marks)
		b Work out the area of the shaded region.	(2 marks)

Complete the PowerPoint, 4.P1-Chp1-Algetraic Expressions

Task 3:

Complete the PowerPoint, 5. P1-Chp2-Quadratics

Task 4:

Complete the PowerPoint, 6. P1-Chp3-Equations and Inequalities

Task 5: Navigating your Casio Calculator